**PHY-Assignment No 1**

1. **Show that using x(t) = cos wt (instead of sin wt) does not change the observable quantities of a harmonic oscillator such as frequency, time period and total energy.**

For a simple harmonic oscillator, the displacement of the oscillator can be represented by the equation:

x(t) = A sin(wt + phi)

where A is the amplitude of the oscillator, w is the angular frequency, t is time, and phi is the phase constant.

Using the identity cos(theta) = sin(theta + pi/2), we can rewrite the equation for displacement as:

x(t) = A cos(wt - pi/2 + phi)

If we define a new phase constant, phi' = phi - pi/2, we can rewrite the equation as:

x(t) = A cos(wt + phi')

So, using x(t) = cos(wt) instead of sin(wt) simply changes the phase constant of the motion, but does not change any of the observable quantities of the oscillator.

The frequency of the oscillator, f, is given by:

f = w/(2*pi)

Since w is unchanged, the frequency of the oscillator is also unchanged.

The time period of the oscillator, T, is given by:

T = 1/f = 2*pi/w

Again, since w is unchanged, the time period of the oscillator is also unchanged.

The total energy of the oscillator, E, is given by:

E = (1/2) k A^2

where k is the spring constant. The amplitude A is related to the maximum displacement of the oscillator, xmax, by the equation:

A = xmax

Substituting x(t) = A cos(wt + phi') into the expression for the total energy, we get:

E = (1/2) k A^2 cos^2(wt + phi')

Using the trigonometric identity cos^2(theta) = (1/2) (1 + cos(2theta)), we can rewrite this as:

E = (1/2) k A^2 (1 + cos(2wt + 2phi'))/2

The average value of cos(2wt + 2phi') over a complete cycle is zero, so the total energy of the oscillator is simply:

E = (1/2) k A^2

which is independent of the choice of sin or cos representation for the displacement x(t).

Therefore, we can conclude that using x(t) = cos(wt) instead of sin(wt) does not change the frequency, time period, or total energy of the harmonic oscillator.

2. **Derive the equations for time period, frequency and energy of a Tortional pendulum.**

A torsional pendulum consists of a mass suspended from a thin wire or rod which exhibits torsional oscillations when displaced from its equilibrium position. The torsional pendulum has the following characteristics:

The restoring torque is proportional to the angle of twist.

The angular displacement is small, and so we can use the small angle approximation of the restoring torque.

Let the moment of inertia of the suspended mass be I, and let the torsion constant of the suspending wire be k. Then the equation of motion of the torsional pendulum is given by:

$I\ d^2\theta/dt^2 = -\ k\theta$

where $\theta$ is the angular displacement of the pendulum.

We can write the solution of this differential equation as:

$\theta(t) = \theta_0 \cos(\omega t + \varphi)$

where $\theta_0$ is the amplitude of oscillation, $\omega$ is the angular frequency, and $\varphi$ is the phase angle.

Differentiating $\theta(t)$ twice with respect to time, we get:

$d^2\theta/dt^2 = -\omega^2\ \theta_0 \cos(\omega t + \varphi)$

Substituting this into the equation of motion, we get:

$-\omega^2\ \theta_0 \cos(\omega t + \varphi) = -\ k\theta$

Dividing both sides by $\theta 0 \cos(\omega t + \varphi)$, we get:

$\omega^2 = k/I$

This equation relates the angular frequency, $\omega$, to the moment of inertia, I, and the torsion constant, k, of the torsional pendulum.

The time period, T, of the torsional pendulum is given by:

$T = 2\pi/\omega$

Substituting $\omega = \sqrt{(k/I)}$, we get:

$T = 2\pi \sqrt{(I/k)}$

This equation relates the time period of the torsional pendulum to the moment of inertia and the torsion constant.

The frequency, f, of the torsional pendulum is given by:

$f = 1/T = 1/(2\pi) \sqrt{(k/I)}$

This equation relates the frequency of the torsional pendulum to the moment of inertia and the torsion constant.

The energy, E, of the torsional pendulum is given by:

$E = (1/2)\ I\ \omega^2$

Substituting $\omega = \sqrt{(k/I)}$, we get:

$E = (1/2)\ k\ \theta 0^2$

This equation relates the energy of the torsional pendulum to the torsion constant and the amplitude of oscillation, $\theta 0$.

Therefore, we have derived the equations for the time period, frequency, and energy of a torsional pendulum in terms of its moment of inertia and torsion constant.