Undergraduate Thesis Proposal Example

Title: First-year undergraduate calculus students: Understanding their difficulties with modeling[1] with differential equations[2].

1. Introduction

Reform movements in the teaching of many disciplines, including calculus (Bookman & Blake, 1996; Douglas, 1986; Hughes-Hallett et al., 1998; Johnson, 1995; O'Keefe, 1995) arose from the growing awareness in the late 1970s and early 1980s that while many students could answer straightforward algorithmic type questions, many left introductory courses with significant misconceptions regarding fundamental principles and an inability to apply what they had learned to non-standard problems (e.g. Seldon, Seldon & Mason, 1994; Peters, 1982). In the teaching of introductory calculus at the tertiary level, the response to this situation has, for example, been to emphasise "depth of understanding rather than breadth of coverage" and to be guided by the "Rule of Four: Where appropriate, topics should be presented geometrically, numerically, analytically and verbally" (Hughes-Hallett et al., 1998). The former emphasis resulted from research showing that an overloaded curriculum encourages students to take a "surface" rather than a "deep" approach to learning (Ramsden & Entwistle, 1981) and the latter from the recognition that identifying links between multiple perspectives is necessary for a "deep" understanding and effective problem solving (Schoenfeld, 1992; Tall & Razali, 1993; Anderson, 1996). Following the "Rule of Four" may also benefit students with differing learning styles (e.g. Felder, n.d.; Bonwell, n.d.).

> Introduction indicates the broad area of the research – reform in the teaching of calculus at the tertiary level – and *starts* to indicate the *significance* of the research in a general way by identifying the significance of the pedagogical issues driving the reform.

Reforms in the teaching of calculus have also been driven by research showing that their early tertiary experiences cause many students to become disaffected with mathematics (e.g. Shaw & Shaw, 1997). This is an important issue to address because student engagement and motivation is fundamental to their taking a deep rather than surface approach to their learning. [Could possibly bring in ideas about "affect" here.]

> A continuation of the goal of identifying the pedagogical issues driving calculus reform.

> A clear link to the previously identified problems, thus helping the writing to "flow".

As a means of addressing both the problems of student disengagement with introductory mathematics at the tertiary level and their taking a fragmented/surface rather than a cohesive/deep approach to their studies (Crawford et al., 1994; Redish, Saul, & Steinberg, 1998), some reformers have proposed exposing introductory calculus students to modeling with differential equations early on in the curriculum (e.g. Smith & Moore, 1996; Jovanoski & McIntyre, 2000). The thinking underlying this proposal is that if students see some practical applications of the calculus ideas they are learning, then that will aid both their conceptual understanding and their level of interest in the concepts being taught.

> A narrowing of the focus of the discussion to a particular aspect of the reform. The reader can anticipate that something to do with this narrower aspect will be the focus of the research to be proposed.

> Signal that a critique of the previous reform idea is about to be delivered.

Although the motivations for the above reform approach seem sound, many reforms have failed in the past however (Mueller, 2001), and if current reform efforts are to be more successful, they need to be based on more than the idealism of their proponents, they need to based firmly on an understanding of the ways students think about and construct mathematical knowledge. In particular, in the case under discussion, teaching experience indicates that most students find modeling extremely difficult, so that even if students are provided with the mathematical models and do not have to derive them themselves, the sorts of things that make modeling difficult for them can also be expected to cause them difficulty

> Thus far the background has provided a direct motivation for a need for reform, but this only indirectly points to a need for some research. This paragraph problematizes a particular reform proposal and by doing so, establishes a *need* for some research. Providing the *motivation* for the research is a key purpose of the Introduction.

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[1] Modeling is the process of determining the mathematical equations which describe a particular process.

[2] A differential equation is a mathematical equation which allows one to calculate how a process varies in time (like population growth) or space (like light being absorbed in semi-transparent water) or both time and space (like a rocket flying to the moon).

even at the level of interpreting a given model. Consequently, the risk when using models even as a motivational tool is that if students find them difficult to understand, they may end up seeing them, as Schoenfeld (1992) puts it, as "cover stories for doing a particular mathematical calculation", thus encouraging students to take a "manipulation focus" rather than a "meaning orientation" to their study of mathematics (White & Mitchelmore, 1996). If this happens, the whole rationale for introducing modeling with differential equations will have failed. Consequently, as not much research into the sources of student difficulties with this area of mathematics seems to have been done, it is important if this reform effort is to have a chance of succeeding that research in this area of student learning be conducted.

Note the use of logical connectors to indicate that an argument is being made: "Although" signaled that a critique of the previous reform idea was about to be delivered, while "consequently" indicates that evidence is being used to draw conclusions rather than just related.

While the previous paragraph identified a need for some research in terms of a potential problem and a gap in the literature, this signals that an explanation as to why a *certain type* of research is likely to be helpful is about to be given.

Specifically, the proposed research is based on the notion that since "[m]any students struggle over the same hurdles in the same sequence in learning the same material … descriptive analyses of conceptual understanding are not only feasible, but likely to be widely applicable" (Trowbridge & McDermott, 1980). This notion is reinforced by the successes of this approach in the reform efforts in the closely related field of physics education (see McDermott & Redish (1999) for a review). Consequently, the main aim of this research is to identify and describe the major conceptual difficulties mathematics students have understanding mathematical models of physical problems which involve the use of first-order ordinary differential equations. This aim is a first step to improving pedagogy in this area because if instructors have a sound understanding of the conceptual difficulties students commonly have, then they can potentially design learning activities and sequences which can more effectively help students surmount those difficulties (e.g. Crouch & Mazur, 2001; Hake, 1998). It is also hoped that the research will aid the development of standardized tests of conceptual understanding, similar to the widely used *Force Concept Inventory* (ref.) and *Mechanics Baseline Test* (ref.) used in physics education research, which can be used to provide an objective test of how effective a particular pedagogical approach is in increasing the conceptual understanding of cohorts of students. Precise hypotheses to be tested in the research are provided in section 3.

The *broad* aims of your research should be a logical *conclusion* to the *arguments* developed in your introduction. More detailed aims / hypotheses usually result from a more detailed analysis of relevant literature.

An identification of the potential *benefits* of the proposed research: "Okay, you are addressing a problem and a gap, but if you discover what you hope to, who will that help and how will that help them?"

2. Previous research

Despite considerable reform efforts involving ODEs, not much research into students' understandings of ODEs appears to have been done. Rasmussen (2001) though, has investigated student understandings of various aspects of solutions to ODEs, including graphical and numerical solutions. One important result from this research is that Rasmussen posited that the switch from conceptualizing solutions as *numbers* (as is the case when solving algebraic equations) to conceptualizing solutions as *functions* (as is the case when solving ODEs) is akin to a "paradigm shift" and is nontrivial for students. If Rasmussen's "paradigm shift" idea is correct, then another paradigm shift which might cause students difficulties in the ODEs context is moving from thinking of functional equations as giving the *amount* of a quantity as a function of time t or position x to thinking of a first-order differential equation as giving the *rate of change* of that amount.

Note that one should *not* simply describe previous findings, but use them for some clear purpose. In this case, the purpose is to use the previous research to generate a new conjecture to research.

A second key result from Rasmussen's (2001) research is that some of the difficulties students had with graphical approaches stemmed from either thinking with an inappropriate quantity and/or losing focus of the intended underlying quantity. This observation may be related to the height-slope confusion previously identified in the calculus literature (Beichner, 1994; Orton, 1984). This result also suggests that initially, students may mix up thinking about the *amount* of a quantity and the *rate of change* of that amount.

Another piece of research in this field is by Habre (2000), who explored students' strategies for solving ODEs in a reformed setting. Of interest for the proposed research, is that despite an emphasis on qualitative (graphical) solution methods in the course, the majority of students interviewed still favoured algebraic approaches over graphical approaches at the end of the course, possibly reflecting the heavy algebraic focus of previous mathematical experiences. The research also suggested that students find it difficult to think in different modes (i.e. algebraic and graphical) simultaneously, which might also help explain why students typically don't use multiple modes to tackle problems. (As reported in Van Heuvelen (1991), typically only 20% of engineering students use diagrams to aid their physics problem-solving in exams, and it has been found that even top students used graphs in only one quarter of their solution attempts in a test with nonroutine calculus problems (Seldon et al., 1994)). Habre's research provides support for Rasmussen's (2001) "paradigm shift" conjecture in that it shows that it takes students considerable time to get used to new ways of thinking about mathematical concepts and they may "cling to" or revert to more familiar, and better practiced approaches and ways of thinking even when these approaches and ways of thinking are not completely appropriate or effective.

An aspect of conceptual understanding not addressed by the above research though, is students' ability in modeling contexts to both interpret in physical terms the various terms of an ODE and to translate from a physical description into a mathematical description. These two abilities are the focus of the proposed research and are of course complementary skills. These skills are important as they are needed for students to reason appropriately about solutions and ultimately to develop the capacities to model mathematically using differential equations themselves.

Although the above aspects of student understanding of ODEs do not seem to have been previously investigated, similar aspects of student understanding have been investigated in the contexts of algebraic word problems and various aspects of calculus problems. Thus for example, it has been found that in algebraic word problem translations, common problems were word order matching/syntactic translation and static comparison methods (Clement et al., 1981). Similarly, student difficulties with correctly distinguishing between constants and variables, and between dependent and independent variables in rates of change contexts has also been identified (White & Mitchelmore, 1996; Bezuidenhout; 1998; Martin, 2000). In addition, research on student understanding of kinematics graphs (Beichner, 1994) and velocity and acceleration (Trowbridge & McDermott, 1980, 1981), reveals that many do not clearly distinguish between distance, velocity and acceleration. It is also known that prior to their development of the concept of speed as an ordered ratio, children typically progress through a stage where they think of speed as a distance (the distance traveled in a unit of time) (Thompson & Thompson, 1994). Since modeling with first order ODEs uses similar sorts of concepts and skills, this raises the question of whether the above-mentioned difficulties are still present at the level of instruction to be considered or in the slightly different context of where students are given the models rather than being expected to be able to determine the models themselves.

Important Note: While it may appear that the review is being organised around the results of individual articles, it is in fact being organised around *themes*:
(i) the difficulties of changing patterns of thinking;
(ii) the difficulty of keeping a track of what quantity one is working with; and
(iii) the difficulties students have with mastering new solution methods.
It only *appears* as though the review has been organised around individual articles because of how little research had been done at the time. It is important that reviews be organised around themes and questions.

Note how a *synthesis* of research results is being made here. An important goal of a literature review is to show how the "pieces fit together.

Identification of a gap or deficiency in the existing literature. If there is no gap or deficiency there is no need to do any research.

But there still needs to be a good reason for wanting to fill the gap. Novelty alone is only half a justification for doing some research.

3. Theoretical framework and hypotheses to be tested

Research into student learning could take a cognitive perspective (refs.), a behavioural perspective (refs.) or a social cognitive perspective (refs.). <Discuss briefly what aspects of student learning each perspective provides insights into and then *explain* why the cognitive perspective will be taken in this research.> The purpose of this section is to identify the theories about human learning and cognition which can help us *understand why* students have the difficulties described in the previous section and will guide hypothesis generation.

In building up a long and complex document, it is often a useful strategy to make notes to yourself of things that need to be done if you are not yet ready to do them. Doing what you can when you can gives you a sense of progress and hence helps you maintain motivation. It is also generally more efficient to get ideas down when they are clear in your head rather than delay while you explore something else because by the time you get back to things you might find that they are not so clear to you anymore!

The basic ontological assumption underlying this research is that many student errors in mathematics are not simply the result of ignorance or carelessness, but are in fact systematic and furthermore common to significant numbers of students across a wide range of contexts (refs.). And because they are systematic, research on groups of students can be used to identify what errors/conceptual difficulties are prevalent in a particular context. Identifying such errors is a first step to improving pedagogy as a knowledge of conceptual problems can then be directly addressed in carefully constructed learning sequences and activities, something which has been found to be the case in numerous instances in the closely related field of physics education research (refs.).

But why is it that clear and careful exposition of a subject is not (necessarily) enough to achieve student learning? Why isn't it (necessarily) correct to assume that if a teacher has taught the material "well" then student errors simply reflect a lack of effort on the students' part or a random "slip" which can only be addressed by telling students to "check their work"? The precise answers to these questions are still a matter of debate and research (Confrey, 1990), though there seems to be general agreement that fundamentally it is because students are not "blank slates" for their teachers to "write upon". Rather, students come to class with ideas and conceptions which may both aid or hinder further learning (see for example, chapter 1 of Ambrose et al. (2010) for an overview). In addition, students do not unproblematically absorb new teachings; what they learn (or fail to learn) is affected by both their beliefs about learning and by how they attempt to make sense of what they are taught. (This is the "constructivist principle" (e.g. Redish, 1999; Elby, 2000).) Furthermore, certain aspects of human cognition can also be expected to cause numerous problems. For example, some errors which could be conceived of as carelessness can be reliably triggered in certain contexts and will be if teaching is not cognizant of the problem. For example, answer quickly the following two very simple questions:

1. *What colour is snow?*
2. *What do cows drink?*

If, like many people you answered "milk" to the second question, you have fallen foul of the downside of the fact that (some aspects of?) memory is associative (refs.). While it is certainly of considerable benefit for the development of expertise to be able to link ideas and concepts together in memory schemas (refs.), such links can at times be inappropriately triggered leading to errors in thinking.

Common failings in human reasoning have been widely studied (e.g. refs.), and Davis (1984) discusses many findings from cognitive science and their implications for mathematics teaching and learning, but for the purposes of the proposed research, Perkins' (1995) theory of default modes of thinking seems to provide the best overall theoretical / conceptual framework [For a PhD level proposal, would need to discuss what alternative frameworks were considered and why Perkins was considered the most appropriate]. According to this theory, the pattern-driven nature of human cognition leads to four default modes of thinking which, *while they serve us well most of the time*, can cause problems in novel situations or familiar situations which have been subtly changed (i.e. the typical sort of situations any student faces). These default modes, giving only the negative side of the mode, listing some studies in quantitative disciplines where such problems have been found, and giving the expected implications for this research, are shown in Table 1.

Table 1. Perkins' (1995) four default modes of human thinking and their predicted implications for the proposed study.

Default Mode	**Implications for study**
Fuzzy thinking: exemplified by a failure to clearly discriminate between closely related terms (e.g. Davis, 1994; Maurer, 1987; Bransford et al., 1999; Reif, 1987; Reif & Allen, 1992); and overgeneralising or having deficient applicability conditions (Davis, 1994; Tirosh & Stavy; 1999; Reif, 1987).	For the proposed research, it is therefore expected that a significant number of students will poorly discriminate between the *amount* of a quantity as a function of time or distance and the *rates of change* of that amount. It is also expected that students will not pay close attention to where initial conditions appear in problem formulation, leading to problems in interpretation.
Hasty thinking: exemplified by too rapidly deciding on a solution strategy or solution on the basis of a superficial examination of the most obvious features of a problem (i.e. trying to pattern match the problem to one seen before) (e.g. Reif & Allen, 1992; Chi et al., 1981; Silver, 1987), rather than on deep processing. This thus represents a weakness in the problem-solving approach taken, that is, it is a metacognitive (refs.) weakness.	For the proposed research, it is not just pattern matching solution strategies that is expected to be a problem, it is expected that the modeling problem contexts will trigger in inappropriate ways, conceptions from students' previous studies of functions. This perhaps explains Rasmussen's (2001) idea of a paradigm shift, though the "paradigm shift" that will be under investigation in this study is whether students have trouble shifting from thinking about equations giving the *amounts* of a quantity to differential equations describing the *rate of change* of the amount of a quantity.
Narrow thinking: related to hasty thinking, narrow thinking also represents a metacognitive weakness in that it is exemplified by a failure to consider alternative perspectives or solution strategies.	This is reflected in Habre's (2000) research mentioned above, but will not be explored in the proposed research.
Sprawling thinking: may be useful when one is brainstorming, but is a problem when it leads one to lose track of what one is doing or "to change horses midstream" (Reif, 1987). This also represents a failure to develop effective problem-solving control and monitoring strategies.	This mode is expected to further contribute to students losing track of whether they should be thinking in terms of amounts or rates of change of amounts at various points in problem interpretation or analysis.

Based on the above-mentioned previous research and a consideration of the implications of Perkins' (1995) default modes of human thinking, the proposed study will test the following hypotheses:

Ultimately the goal of the review has been to identify and justify new hypotheses to be tested.

1. That both poor discrimination between the concepts of an amount which varies with distance or time and the rate of change of that amount, and a reversion to thinking about the amount of a quantity rather than its rate of change will cause students to think that quantities on the right hand side of a first order ODE should be multiplied by the independent variable (i.e. distance x or time t).
2. Due to a combination of "direct translation" and a failure to pay attention to the distinctions between the amounts of a quantity and the rate of change of that quantity, significant numbers of students will tend to:
    a. put the initial conditions for a problem into the ODE which describes the modeled phenomenon; and

b. interpret constant terms in an ODE as representing an initial amount rather than as a constant rate of change of the dependent variable.

3. That a significant number of students will either not be aware that the units of the terms of any equation need to be homogeneous or will not think to use that condition to check their answers / reasoning.

4. Expected outcomes and their pedagogical implications

The expected outcome is to be able to describe in detail the most commonly occurring conceptual difficulties students have with modeling with differential equations for a wide range of modeling situations. These outcomes will help instructors in this area develop learning activities which will help students overcome these conceptual difficulties, and thereby not only improve the learning experience of students, but also increase the chances that reform efforts using modeling with ODEs will prove fruitful rather than another disappointing fad. In particular, if the research supports the various hypotheses, various exercises which draw students' attention to the discriminations they need to make are fairly evident and could be tested in future research as to how effective they are in helping students make the appropriate paradigm shifts. [A PhD project could thus be a multi-stage project where the first stage is as discussed in this proposal, with later stages testing the effectiveness of different pedagogical approaches in achieving gains in students' conceptual understanding.]

5. Methods

Materials and their justification

In order to determine how widespread the various misconceptions and difficulties might be, the whole student cohort will be given diagnostic quizzes consisting of a mixture of multiple choice and short answer questions. The multiple choice questions will include a number of problems typical of the ones seen in class with distractors chosen to reflect the hypotheses given above and other likely student errors (e.g. making a sign error).

The reasons for choosing to use diagnostic multiple choice questions are as follows. First, while the differential equations which describe a scenario will have been developed in class, students will not have been expected to be able to develop their own DEs as the focus of the course will be on providing a motivation for doing integrals. Consequently, what is of interest is whether they can "read" and understand a DE. This could be assessed by asking students to describe in words what they understand a DE to be saying, but as students may not be able to articulate clearly what they are thinking this might not be a very successful approach for a written quiz (we might get a lot of blank responses for example) and so this approach will be left to individual interviews as described below. Consequently, it is proposed in the written tests to assess students' ability to read and understand a DE in lower level ways by asking them to determine which of a range of choices accurately describes a given scenario and by asking them to identify the units of each of the terms in a DE (see below).

Note that proposals (and theses and research articles in general) should not just *describe* the methods use, but also provide a *justification* for the methods used. The reviewer wants to be convinced that the methods have been well thought through and there are good reasons for conducting the research in the ways proposed.

As stated above, in questions asking students to identify which of a range of possibilities is the correct DE describing a given scenario, distractors which reflect various hypotheses about the likely "bugs" in student thinking will be provided. It is hoped that the prevalence of such bugs will then be able to be determined and so will provide insights into common issues with student thinking. Giving students instructor determined DEs to choose from might also help control for many other variables which might influence a student's free response and which would therefore make interpretation of their answers more difficult. Finally, students should also be able to do multiple choice questions faster than other types of questions and hence a wider range of problems can be covered in the time available. This is important because giving students a range of questions may help determine how context specific student thinking might be. <Needs some references backing up this sort of approach and why it might be expected that student thinking might be context specific.>

Although they have many advantages, multiple choice questions also have a range of disadvantages. First, forced responses may cause students to think differently than they would have done if left to their own devices (refs.). In the present case, this problem might arise if the presence of some choices generates questions or confusions in students' minds which would not have occurred to them in the absence of those distractors. In addition, students may get the right answer for the wrong reasons or choose a distractor for reasons different to the hypothesized ones. Consequently, there is a need to validate any conclusions drawn from the MCQ diagnostic questions using a process of triangulation (refs.). This will be done in two ways. First, the diagnostic quizzes will have, in addition to the MCQs, open-ended questions asking students what they think the units of various terms in an ODE are. Second, representative students will be asked to explain their choices in a one-to-one interview and work some further questions using a "think aloud" protocol (refs.).

Any given method generally has weaknesses. It is important to demonstrate that you are aware of these weaknesses and have thought through how you will compensate for them. In addition, you need to convince the reviewers that despite the methodological weaknesses, sound results are still likely to be obtainable.

<Now need to explain why getting students to give the units of terms in an equation is likely to help, and why it might also be misleading and so also needs cross-validation. The reasons for using a think aloud protocol need to be given and how that will help with cross-validation and how it too may be misleading. It will also need to be explained and justified as to how the students who will do the one-to-one sessions will be chosen.>
Both to confirm the researcher's interpretation of student answers and to clarify responses which either cannot be interpreted or may have multiple possible interpretations, taped one-to-one interviews with selected students will also be conducted. During these interviews, "think aloud" protocols will be used as the mathematical problem-solving literature has shown that post-hoc explanations of reasoning often do not reflect accurately what thinking actually did occur (Davis, 1984; Schoenfeld, 1992).

Note again how the method is not just being described, but the reasons for using it are also explained.

Participants
This research will use a convenience sample of first-year officer cadets studying introductory calculus in both the Bachelor of Science and Bachelor of Engineering programs at the Australian Defence Force Academy (ADFA). An issue raised by such a sample is whether the findings with such a particular group are generaliseable to students studying mathematics at "regular" universities. That is, will the findings have external validity? It is believed that this group does *not* in fact pose any significant problems for external validity for the following reasons. First, although the students are officer cadets, the degree programs they are studying are offered through the University of New South Wales and hence they are studying the same sorts of maths that students studying introductory calculus at any Australian university would be studying. In addition, because ADFA is in fact more selective than the average Australian university, it can be expected that the officer cadets will not be any weaker, on average, in mathematics than their peers at regular universities; while the very top mathematics students may be missing from this cohort, so too will those at the bottom end. Consequently, if anything, any conceptual difficulties raised by this research are likely to be somewhat more

prevalent and pronounced in the general university population. Finally, there is in fact an advantage to doing this study at ADFA. As the officer cadets studying there come from all over Australia, the possible influence of different State-based education systems can potentially be investigated. This would not be possible at regular universities where almost all domestic students could be expected to have had the same state-influenced school curriculum.

Procedures & ethical considerations
<Need to explain and justify the timing of the quizzes and the interviews. >

… An ethical issue that is likely to arise in the individual interviews is that students might feel quite tense about working in front of an instructor and might get distressed if their lack of understanding is exposed or they get themselves confused while working on a question. <Need to explain how will handle such a situation if it arises. Also need to explain to participants at the beginning of an interview that if at any time they wish to cease participating in the interview, that they are free to leave and that there will be no consequences for doing so. This suggests that the interviews would be best done, if possible, by someone other than the course instructor.>

Another ethical question raised by this research is that if we can anticipate what students' difficulties will be, aren't we ethically obligated to address them in our instruction? A response to this is that since the curriculum is already quite full, taking extra time to address issues which may or may not be quite prevalent, is unjustified and may be perceived by students as treating them as "dummies". Furthermore, the research needs to be done so that instruction is evidence-based and targeted at the right places rather than based on conjectures which may or may not be valid and which may lead to a "scattergun approach". <There are more arguments which could be brought in here.>

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